For ages 10-15

MATH MIRACLES

Sixteen Amazing Math-e-Magical Tricks

By
Matt Baker
Joshua Jay
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VANISHING INC.

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INTRODUCTION

Math can be used in all sorts of fascinating ways — to calculate the speed of a soccer ball or to measure the length of an allosaurus from nose to toe, or to compute the number of steps required to walk from Delhi to Agra. We typically use numbers for functional purposes—to solve problems or determine information. But it never crossed my mind that math could be used to amaze people until I became a magician. And I was never truly amazed by mathematics until I began studying the secrets of magic.

Math Miracles is filled with amazing magic that will astound your audience: tricks with photos, emojis, fingers, and even drinking glasses. But what's most impressive about this collection of tricks is that each one relies on secret mathematical principles. It may look like you're doing magic, but in each case you're actually doing math.

Learning the steps of these magic tricks and why they work is a fantastic way to learn about concepts in geometry, algebra, and topology, as well as more advanced principles such as what mathematicians call "error-correcting codes." Each trick relies on a different mathematical principle, and every trick has been adapted by clever magicians to obscure the principles on which it relies. I love this secret intersection of magic and math. "Science and art sometimes can touch one another," wrote artist M.C. Escher, "like two pieces of the jigsaw puzzle which is our human life, and that contact may be made across the borderline between the two respective domains."

If you're counting the many great reasons to learn about math through magic, here's one more. According to an academic study published in April 2021, learning magic tricks enhances a student's creativity. A group of ten- and eleven-year-old students in London were asked to take tests measuring creativity before and after learning skills like art and...magic tricks. Students who were taught magic tricks performed significantly better in tests of creative fluency and originality than those who were taught an art lesson. So while these tricks will definitely improve your math skills, they may also help you be more creative.

I have invited two fellow magicians to coauthor the material in this book. Matt Baker is a Professor of Mathematics at the Georgia Institute of Technology. He is a mathematician by day and wickedly clever magician by night. Andi Gladwin is a UK-based professional magician, and in 2021 he managed to devise a mathematical magic trick so deceptive that it fooled master magicians Penn & Teller on their television program *Fool Us*. The three of us have scoured the world for the very best "math-e-magical" tricks, and in each case we've not only described how to perform the tricks but have also explained the math behind each one. You can skip around if you like, keeping in mind that the tricks get more difficult as the book progresses. Younger readers will find favor with the first few tricks, and older students may want to skip right over to "The Missing Digit" on p. 28.

The British mathematician Marcus de Sautoy wrote, "Mathematics is a place where you can do things which you can't do in the real world." And this is also *exactly* how I would define magic. So I invite you now to leave the real world and enter the realm of mathematics — a place where you can, for a few moments in front of your audience, do the impossible.

Joshua Jay
New York City
2021

ABOUT US

"We" are three magician friends who share two passions: we *love* magic and we also *love* numbers. When we discovered Agasyta, we thought that working with them on a curriculum that teaches math principles *through* magic tricks would be an unusual, exciting way to learn about math.

Joshua Jay has performed magic in over 100 countries. He is a former champion in sleight-of-hand magic and holds the Guinness World Record for card tricks. He has written four best-selling books on magic, including *MAGIC: The Complete Course* and *How Magicians Think*.

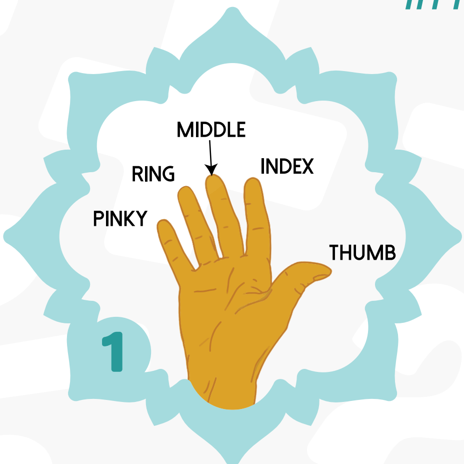
Matt Baker is a Professor of Mathematics at the Georgia Institute of Technology and also an accomplished magician. He has published three math books and over 45 research papers and is a Fellow of the American Mathematical Society. His debut book on magic, *The Buena Vista Shuffle Club*, was a critical and commercial hit.

Andi Gladwin is a professional magician based in the UK. Gladwin has performed numerous times on television, and recently fooled Penn & Teller on their hit show, *Fool Us*.

CONTENTS

FINGERTIP MINDREADING

You know that magicians have special hands. In this case, your hands are the props. You will correctly predict a chosen finger. This trick works great one on one, but it's also fun to do in front of a big group.



1. Begin by asking everyone to hold up their right hand so that their palm is facing their face. You can follow along with this, too. Your right thumb should be on your right and your right pinky finger should be all the way on the left. If needed, you can clarify where the index, middle, ring, and pinky fingers are on a hand, and don't forget to mention the thumb as well (**illustration 1**).

2. Explain to the audience that they will soon have a free choice to make any *five* moves. A move counts as moving from one finger to another. You can move in any direction, but you can't skip over any fingers. You might demonstrate by touching your left index finger to your right pinky finger, and then touching your right ring finger, and then moving to your right middle finger, and then back to your right ring finger,

and so on. Once people understand how to move from one finger to the next, you can proceed **(illustration 2)**.

3. Turn your back to the audience so that you can't see everyone holding up their right hands. Ask everyone to begin on their right pinky finger. Now encourage them to make any five moves they like, back and forth. At the end of these five moves, ask everyone to keep touching their chosen finger **(illustration 3 and 4)**.

4. *"Hmm. Each of you has made some interesting choices. But I get the feeling you haven't landed on your pinky finger. So let's eliminate that finger by folding it down."* Indeed, it's impossible for anyone to land on their pinky finger. Allow everyone to fold down their pinky finger **(illustration 5)**. *"I also get the impression you're not on your thumb. So let's fold that down, too."* Again, it's mathematically impossible to make five moves and end on the thumb, so allow everyone to fold that down, too **(illustration 6, next page)**.

5. *"Let's all do another single move...now."* Allow everyone to perform exactly one move. Regardless of where they began, one move forces them onto the middle finger. For the first time in this trick, you can be certain absolutely everyone (who followed your directions correctly) is now holding their middle finger. "Let's eliminate the ring finger now." Encourage everyone to fold down their ring fingers **(illustration 7, next page)**.

6. *"It's time for one last move. Do that now."* Everyone will be forced to move from the second finger to the first. *"And I get the impression that you're not on the middle finger right now. So fold it down."* Allow the participants to fold down their middle fingers.

7. Time to amaze everyone. *"Despite having an entirely free choice at every stage of this trick, I believe you've all ended up on one finger in particular. I want you to use that finger and angle it at me right now!"* Turn around and point your index finger at everyone in the audience. You will see an audience full of index fingers, pointing right back at you. Somehow, you correctly predicted which finger they would choose **(illustration 8)**!

The question is, *how* did you do it?

WHY IT WORKS

This trick is all about **parity** — in other words, odd versus even.

For the purposes of this explanation, let's number the fingers as follows:

Thumb = 1, Index = 2, Middle = 3, Ring = 4, Pinky = 5 **(illustration 9)**.

Each time one of your spectators makes a move, the parity of the finger they're pointing to changes (in other words it either goes from odd to even or even to odd).

They start on the pinky (#5), which is ODD. So after five moves (Step 3), they must end up on an EVEN-numbered finger (index or middle). That's why we can safely eliminate the pinky (#5) and thumb (#1) at this point. (We could also eliminate the middle finger now if we wanted to, but that would ruin the rest of the trick.)

Now there are just three fingers remaining (#2, #3, and #4), and you know the spectators are pointing to an EVEN finger (either #2 or #4). The next move (Step 5) must take the spectator to an ODD finger, but the only odd finger left at this point is #3. So at this stage everyone who followed your instructions will be pointing to their middle fingers.

At this point you eliminate the ring finger (#4) and you ask the spectators to make one more move (Step 6). Since there are just two fingers remaining (#2 and #3) and they start out on #3, they must end up at #2. You can now eliminate #3 and everyone will be left with just their index fingers pointing out!

FOLLOW THE MONEY

In this amazing stunt, you ask someone to hide a banknote under one of three inverted cups, and then you invite the participant to mix up the cups while your back is turned. When you turn back around you're immediately able to...follow the money!

1. This trick works because one of the three cups is secretly marked in a way only you can see. We suggest you use plastic cups and that you mark one with a small black dot on the bottom**(Illustration 1).** Nobody else is looking for a dot, and the cups will appear otherwise identical **(Illustration 2).**

2. Invert the three cups on the table in a row and situate the marked cup on your right. Now ask a participant to remove a banknote and crumple it into a ball **(Illustration 3).**

3. Turn your back and ask the participant to hide the banknote under any one of the three cups. They really have a completely free choice.

4. *"Perhaps you think I memorized the exact position of those three cups on the table,"* you say. *"And since you only touched one of the cups when you hid the money, let's switch the positions of the two empty cups."* Allow the participant to switch the positions of the two un-chosen cups.

5. *"Now you've hidden the money and mixed the cups around. It's my job to figure out where you hid the money."* When you turn around, one quick glance at the row of cups will tell you everything you need to know.

6. There are three simple rules to remember:

a) If the marked cup is still on the right, you know the money is under this "right" cup **(Illustration 4).**

b) If the marked cup is on the left, the money is under the center cup **(Illustration 5).**

c) If the marked cup is in the middle, the money is under the left cup **(Illustration 6, next page).**

7. Lift the correct cup to display the money. We think this trick is so impressive that you should be able to keep the money, but your participant may disagree.

3

4

5

WHY IT WORKS

This is an example of a principle magicians will often use. *You* are secretly aware of one extra piece of information your audience doesn't know. In this case it's that one of the cups is marked. By knowing the starting and ending location of a particular cup, it's easy to discern the three possible outcomes and correctly find the money. The trick seems impossible because the audience doesn't know that you've marked one of the cups. If you follow the procedure in each of the three cases (the bill starts in the right, center, or left cup, respectively), you'll understand visually exactly why it works!

6

ANIMAL IMAGERY

In this trick you predict an animal and a country merely thought of by a participant. And it all begins with a thought-of number. Follow along below and you'll fool yourself!

Here are the instructions which you give to your participant:

1. Think of any number between 1 and 20.

2. Now *double* your secret number.

3. Add 8 to the result.

4. Now divide your total by 2.

5. Next, subtract your original thought-of number from your new total. This will be your secret random number. We hope you will agree it's not a number even YOU thought you would land on a moment ago.

6. Now let's convert your number into a letter. A = 1, B = 2, C = 3, and so on. Thinking of a letter? Good.

7. Think of a country that starts with this letter. Any country in the world.

8. Think of the second letter of this country. Now think of an animal that starts with this letter (the second letter of your thought-of country).

9. So you're thinking of a country and an animal... take a look at my prediction!

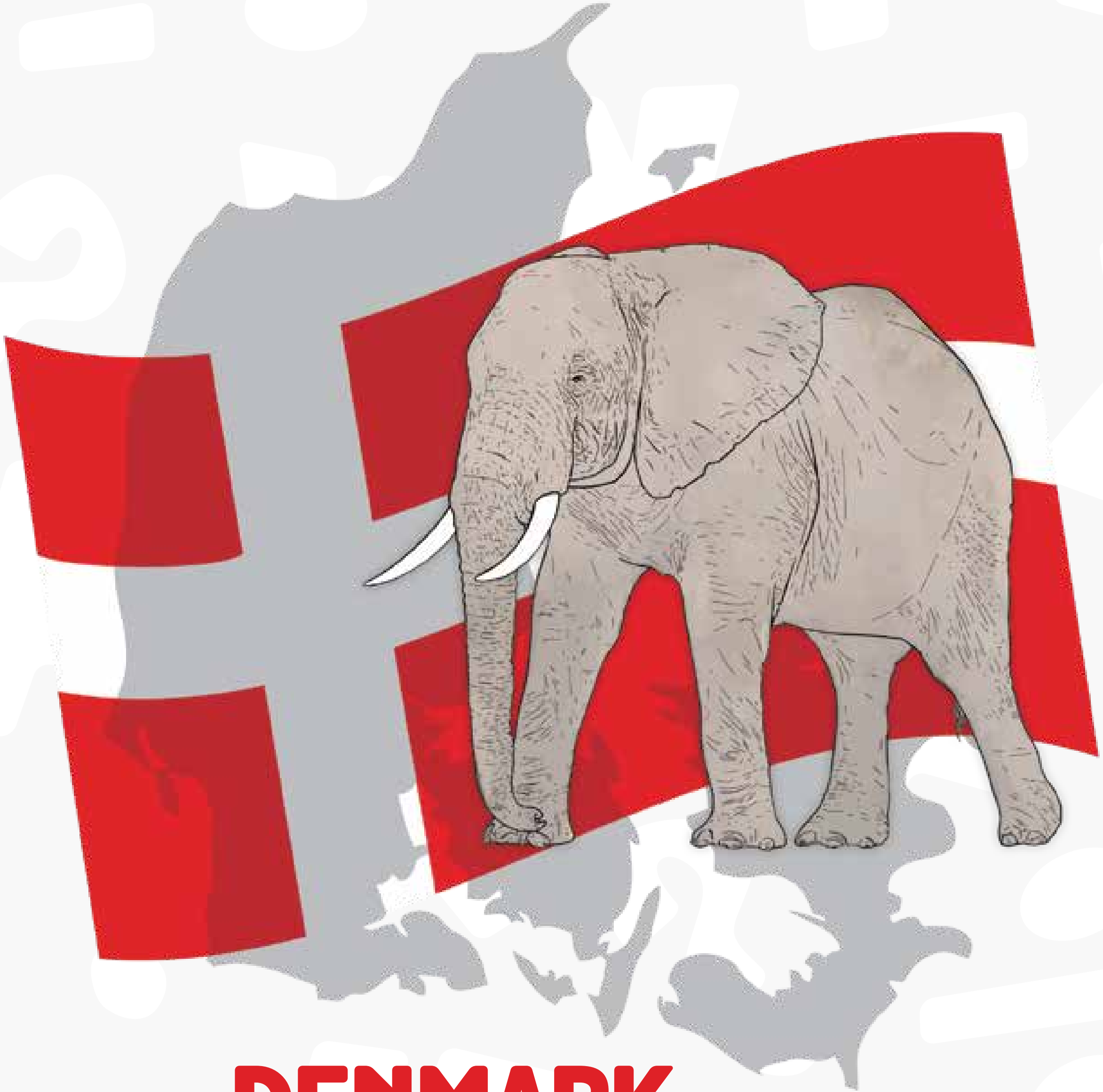
You can either use the artwork on the page as your prediction, or you can draw your own picture.

WHY IT WORKS

It seems impossible to be able to guide someone's thoughts from a freely chosen number to a specific animal and country. But that's exactly what you do. At the end of Step 5, your participant will always be thinking of the number 4. And we can use algebra to figure out why...

Let's use the unknown quantity X to denote the number they think of in Step 1. When they double it in Step 2 and then add 8 in Step 3, they're now thinking of $2X+8$. And when they divide this by 2 in Step 4, they are thinking of $X+4$. So, when the participant subtracts their original number X from the current total of $X+4$, they are always left with $(X+4) - X = 4$.

There is an element of risk involved in Steps 6-8, but this is what makes the trick memorable and impressive. There are other countries that start with D (the Democratic Republic of the Congo and Djibouti are two examples), and there are other animals that start with E (for example eagles and eels). But overwhelmingly, the most common choices are Denmark and Elephants, so this is your safest bet.



DENMARK

MATH PUZZLES

These math puzzles are fun to use on your friends. They aren't magic tricks, but they are just as sneaky as the tricks you have already learned!

555

Here's the challenge: you can only add just one straight line to the sum below to make it correct:

5 + 5 + 5 + 5 = 555

The rules are that you can't use the "not equals" sign (≠) and you can only add one line. Can you figure it out?

Answer: This is sneaky! You add a diagonal line to any one of the + symbols to turn it into a number 4. The sum is now 545 + 5 + 5 = 555.

TEN TO ONE

This one is fun to do as you don't need to write anything down! Quickly — we want you to count from ten to one backwards. Go!

Did you count 10, 9, 8, 7, 6, 5, 4, 3, 2, 1? Then you got it wrong!

Answer: This math puzzle isn't about math at all – it's about thinking carefully about the words used. 10 to 1 backwards means that you should start counting at 1 and end on 10. If we asked you to count from 1 to 10 backwards, you would start at 10 and count backwards to 1, but that wasn't what we asked. Sneaky!

1240

Write the sum **(on the right)** on a piece of paper. Can you remove two rows of the sum and still make it add up to 1240?

318
303
300
104
215
1240

Answer: This one is almost impossible for your friends to guess! You need to fold the paper over between the numbers 303 and 300, to turn those two rows into the number 707 **(Illustration 1)**.

MAKING SOMEONE VANISH

Did you know that it is possible to make someone vanish into thin air using mathematics alone? In this simple trick, we use some clever geometry to make it look like a picture of Neelu has vanished from a banknote. The best part is that you will be able to perform the trick on yourself. Follow along!

1. Pop out the pieces of play money, cut along the lines and line them up on the table to create the banknote **(Illustration 1 and 2)**.

2. Remove the small piece of Neelu's face and place that piece in your pocket. We're going to make her disappear **(Illustration 3)**!

3. Jumble up the pieces that are left on the table and turn them over. You'll notice that the back of the play money is a slightly different shade of purple **(Illustration 4, next page).**

4. Build the banknote again. Amazingly, it fits perfectly together without the missing piece. Neelu has vanished **(Illustration 5, next page)**!

You can add Neelu back in and turn the pieces over again. The play money can be built again, exactly the same. Amazingly, you can even measure the note (measuring *inside* the border) and it's exactly the same size with or without the Neelu piece!

WHY IT WORKS

The secret to this illusion is pure geometry. The pieces seem to fit back together in exactly the same way when Neelu has been removed, but that's not actually true: the rectangular shape is actually a little smaller when the piece is missing.

To make this work, the pieces actually have a different orientation on the back. The outer corners of the four pieces point inward, instead of outward, like you might expect. And so, the bill shrinks on the length and width to the exact area of the missing piece.

To prove this, put the pieces together with Neelu showing upward and draw around the play money on a piece of paper. Then, perform the trick and you will notice that the new banknote is actually smaller than the original.

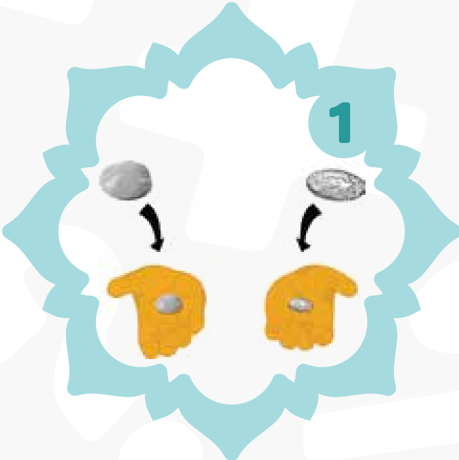
There's one more sneaky part of this secret: that we allowed you to measure the banknote during the performance. The secret there is that we said you could measure *inside* the border. With a careful design of the bill, we were able to add to the illusion by making the design on the back of the note have a slightly larger border than the front. This is proof that magicians will take every opportunity to fool you.

WHICH HAND?

In this trick, we will use logic to decipher which of your friends is lying to you, and which is telling the truth.

You need two different objects; let's say a coin and a pebble (but it can be any two objects that can fit inside your closed hand).

1. Place the two objects on the table and invite two friends (let's call them Dhruv and Anju) each to pick up one object. Turn away from them while they do this. Say that whoever picks up the coin should put it in their left hand, and whoever picks up the pebble should put it in their right hand **(Illustration 1)**.



2. Ask them to close both hands into fists so that you don't know who has the coin and who has the pebble, or which hand the objects are in. When they have done this, you can turn back around.

3. At this point, you genuinely have no idea which object is where. But now we'll use a simple logic puzzle to find out who has which object and which hands they are in. Explain that you are going to ask each of them a question, but there's a fun twist: Whoever has the coin should lie when they give you their answer, and whoever has the pebble should tell the truth.

4. Ask Dhruv, *"Is the coin in your right hand?"*

5. Once he answers, tell Anjali that, to make things even more challenging, she should put her hands behind her back and may secretly switch which hand holds the object IF SHE CHOOSES TO.

6. Ask Anjali, *"Did you switch hands?"*

7. Believe it or not, you now have all the information you need to reveal who has what in which hand! For example, you might now exclaim, *"Anjali, open up your left hand and show the coin, and Dhruv, open up your right hand and show the pebble!"*

HOW TO DO IT

- If Dhruv answers yes to the first question, he has the coin in his left hand. If he answers no, he has the pebble in his right hand.

- Whichever object Dhruv does NOT have is the one Anjali has. If she answers yes to the first question, the object is in her left hand. If she answers no, the object is in her right hand.

In other words: A yes answer always means LEFT, and a no answer always means RIGHT. A yes from Dhruv means he has the COIN, and a no means he has the PEBBLE.

WHY IT WORKS

If Dhruv has the coin, it is in his left hand and he is a liar. So if you ask him, *"Is the coin in your right hand?"* he will answer yes.

And what if Dhruv has the pebble? Well, in that case he's a truth teller, so if you ask him, *"Is the coin in your right hand?"* he will answer no.

Therefore, after the first question, you know whether Dhruv has the coin or the pebble, which means you ALSO know which object Anjali has. Moreover, because of the original instructions, you know which hands they're holding them in as well!

The second question is there just to make things a little more interesting and distract your friends from thinking about the FIRST question too much.

Suppose Anjali has the pebble. If she answers yes to the second question, it means she switched, so the pebble is in her left hand; if she answers no, it means she didn't switch, so the pebble is in her right hand.

On the other hand (pun intended), suppose Anjali has the coin. If she answers yes to the second question, it means she did NOT switch, so the coin is in her left hand; if she answers no, it means she DID switch, so the coin is in her right hand.

MYSTERIOUS MAGIC SQUARE

Many magic tricks are older than you might think. This one was discovered inscribed in Khajuraho, India, in the eleventh or twelfth century, but is as impressive today as it has ever been. It is an amazing display of mental dexterity where you are able to make incredibly difficult calculations in just a few seconds.

28	3	2	13
5	10	23	8
9	6	7	24
4	27	14	1

Imagine that you told us that your favourite number was 46. In just ten seconds, we could demonstrate some amazing mathematics by writing this grid **(on the left)** down on a piece of paper.

So, why is that amazing? Try adding up any of the rows... each one adds up to your favourite number, 46. Now try the columns... also 46! Try the four corners, or the four squares in the middle, or any of the four quadrants (for example the upper left-hand quadrant gives 28 + 3 + 5 + 10 = 46). Even the diagonals add up to 46. See if you can find still other ways of making 46 with your grid!

The good news is that you don't need to be a math genius to make this work. You just need to remember a few number patterns and do some very simple calculations.

1. Start by drawing a blank four-by-four grid on a piece of paper. You will shortly fill that grid with numbers **(Illustration 1)**.

2. Ask someone to give you a number **(Illustration 2)**. It could in principle be any integer (even a negative one!), but the magic square you end up with will look more elegant if you restrict their choice a bit, say between 25 and 50. Let's call the number they name N.

3. Immediately get to work filling in the square. You will need to remember these numbers (and the locations of the four empty squares) before your performance as they will be the same each time you perform the trick **(Square 1)**.

Notice the nice "snake-like" patterns made by the numbers 1 through 4 and 5 through 8. These patterns will help you remember where all the numbers go **(Square 2, next page)**!

4. Now for the sneaky bit. You will complete the four blank squares by first subtracting 25 from the named number N (we'll call the result X, so X = N-25). You then insert the following numbers into the blank squares **(Square 3, next page)**:

1

	3	2	13
5	10		8
9	6	7	
4		14	1

2

	3	2	13
5	10		8
9	6	7	
4		14	1

3

X+7			
		X+2	
			X+3
	X+6		

4

	3	2	13
5	10		8
9	6	7	
4		14	1

For example, if the named number was 40, the final grid would be **Square 4 (bottom).**

The quicker you are able to write down the numbers, the more impressive the trick becomes.

5. You can now show everyone that all of the rows and columns add up to the named number. To make it seem even more impressive, you could have your audience use a calculator to add up the numbers. If they need to use a calculator to add up the numbers, you must clearly be a mathemagical genius!

WHY IT WORKS

If you take a look at the partially completed square in Square 1, you'll notice that for each of the four blank squares, the sum of the other three numbers in that square's row is equal to the sum of the other three numbers in that square's column. The same goes for that square's quadrant (and diagonal, when relevant).

For example, focusing on the blank square at the upper left-hand corner, we see that both the top row and leftmost column add up to 18. So does the upper left-hand quadrant (minus the blank square), and the diagonal which runs from the upper left down to the bottom right. So if you write the number N-18 (which is the same as X+7) in the upper left-hand square, all of these totals will become (N-18)+18 = N.

The same thing happens for the other three blank squares, except (reading from left to right) the totals are 19, 23, and 22, respectively, so the missing numbers are N-19 = X+6, N-23 = X+2, and N-22 = X+3 (see the top square in Step 4).

The resulting grid, once you've put numbers into all the squares, will be:

N-18	3	2	13
5	10	N-23	8
9	6	7	N-22
4	N-19	14	1

We've already seen why the rows, columns, quadrants, and diagonals in this square all add up to N. But the square is constructed in such a way that we also get N when we add up the four numbers in the middle square, or the four corners (and there are other patterns still — can you find them?).

THE MISSING DIGIT

This is a trick you can do any time you or your spectator has a calculator or smart phone with a calculator app handy. It's a practical and baffling demonstration.

For the purposes of explaining the trick, we'll assume that your friend has a cell phone with a calculator app on it.

1. Have your friend take out her cell phone and open up the calculator app.

2. Ask her to type in a random digit from 1 to 9 and then hit "times" (multiplication key).

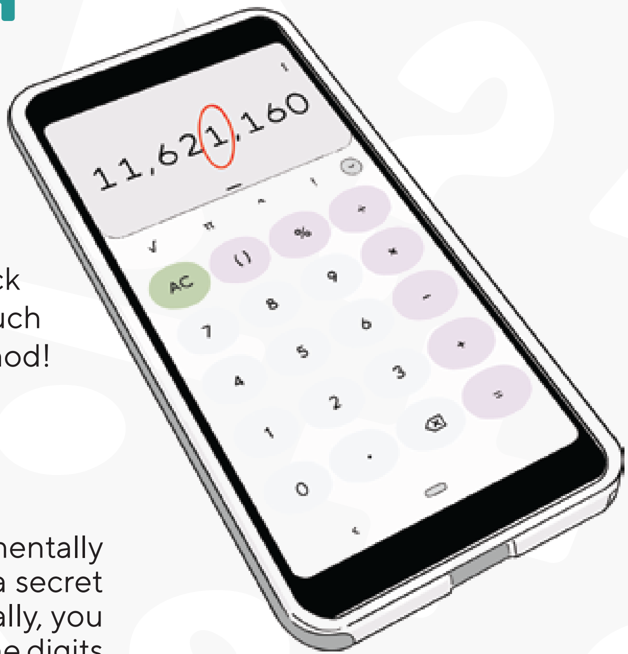
3. Now ask her to type in another random digit from 1 to 9 and hit "times" again. Tell your friend to keep repeating this procedure until she has a number with at least 8 digits.

4. Ask your friend to mentally 'circle' one of the digits in her number, but ask her not to circle a zero, *"since that can be confusing."* Now ask your friend to concentrate on her mentally chosen digit.

5. Say to your friend, *"I'd like you to slowly and clearly read aloud all of the digits in the random number you're looking at except for the one you mentally circled — just skip that* *one. You can recite the digits from left to right or right to left, as you wish, but when you get to your mentally chosen digit, just skip over it and continue reading with the next one."*

6. After your friend has finished calling out digits, you tell her which digit she mentally circled!

A LITTLE CONFESSION



OK, we'll let you in on a little secret: the trick doesn't *always* work. But it works more than 92% of the time! (You'll understand more after you read the explanation below.) In practice that's usually good enough, and the fact that the trick doesn't always work will make it that much harder for your friends to figure out the method!

HOW TO DO IT

How do you figure out your friend's mentally selected number? You're going to perform a secret calculation known as "casting out 9s." Basically, you (silently, and in your head) add together all the digits which your spectator reads out loud, but each time you get a running total of 10 or more you add together the digits of THAT number. In this way, your running total is always a single-digit number. The digit your friend mentally circled will be 9 if the final total is 9, and otherwise it will be **9 minus the final total**.

An example will probably help a lot here. Suppose your friend reads the following digits out loud: 1, 1, 6, 2, 1, 6, 0. You keep a “running total” as follows:

1 + 1 equals **2**
2 + 6 equals **8**
8 + 2 equals 10, but we replace 10 by the sum of its digits: 1 + 0 = 1, so our new “running total” is **1** rather than 10.
1 + 1 = **2**
2 + 6 = **8**
8 + 0 = **8**

Our final total is 8, and therefore the mentally circled digit is 9 - 8 = **1**.

WHY IT WORKS

There are actually two different mathematical principles at work here, and it’s the **combination** that is especially sneaky. The first is:

Principle #1: *With very high likelihood*, the random number that the spectator ends up with after all the multiplications *will be a multiple of 9*.

Indeed, all the spectator needs to do is either type in the number 9 once, or some combination of 3s and 6s twice, and she will automatically end up with a multiple of 9. Because of this, it can be shown that there’s over a 92% chance that the spectator will end up with a multiple of 9. In other words, the trick will fail less than 8% of the time.

The second idea at work here is:

Principle #2: If you add together the digits in any multiple of 9, the result will also be a multiple of 9.

So in the example above, if we assume (by Principle #1) that our friend’s mentally selected number (call it X) is a multiple of 9, we know that 1 + 1 + 6 + 2 + 1 + 6 + 0 + X is a multiple of 9.

But $1 + 1 + 6 + 2 + 1 + 6 + 0 = 17$, so this means that $1 + 7 + X$ is ALSO a multiple of 9. Since $1 + 7 = 8$ and $8 + X$ is a multiple of 9, X must be equal to 1.

“Casting out 9s” has many applications in real life, for example to error-detecting codes such as the Hamming Code (which will be used in the trick “Build Your Own Lie Detector” later in this book). It is believed to have originated in India; the first known description appeared around 1150 in Bhaskara II Acharya’s book *Lilivati*. (This book was also the first known place where zero was treated as a number in its own right.) The result of the mental calculation that you do in this trick is known as the *digital root*, *Beejank*, or *Navaseh* of the digits that your friend reads out loud to you.

PREDICTED CHANGE

This is an amazing trick which can be done with any (odd) number of coins of any denomination. For the sake of concreteness, we'll use eleven coins. You can either borrow the coins from your friend, or you can bring a bunch of coins yourself, hand them to your friend, and ask her to choose any eleven.

1. Ask your friend, Uthara, to place the eleven coins in a row in whichever order she prefers **(Illustration 1)**.

2. Tell Uthara that the two of you will play a game, and you're going to try to predict the outcome in advance. Write down a prediction on a piece of paper, fold the paper in half so the writing can't be seen, and place the prediction to the side. (We'll explain later how to figure out the prediction you're going to write down.)

3. Explain the rules of the game to Uthara. She gets to go first, and then you and she will alternate turns. Whoever's turn it is, that player will always remove one of the coins from the end of the current row of coins and place it into a pile of coins in front of them.

4. When the game is finished, Uthara will have six coins in front of her and you will have five. Ask her to total up the value of all the coins she chose for herself, and then to do the same for your coins. Let's say she ended up with 76 rupees and you ended up with 42 rupees.

5. Ask your friend to open up the prediction and read it out loud. It will be 100% correct! In the above scenario, if your name is Nehal, it would say, *"Nehal will win with exactly 76 rupees and Uthara will end up with just 42."*

HOW TO DO IT

Imagine that the eleven coins are numbered 1-11, reading left to right (from your point of view). What you do while pondering your prediction is to secretly add up the values of all the coins in odd-numbered positions (1, 3, 5, 7, 9, and 11), and then do the same for all the coins in even-numbered positions (2, 4, 6, 8, and 10). Remember these two numbers (let's call them A and B, respectively). If A is larger than B, your prediction will be: "Uthara will win with exactly A rupees and Nehal will end up with just B." If B is larger than A, the prediction will instead read: "Nehal will win with exactly A rupees and Uthara will end up with just B." (If it so happens that A=B, then the prediction should instead read: "The game will be a tie: we will both end up with A rupees!")

If you find it difficult to do the required math in your head, you can also lay out your own set of coins in a pattern that you've memorized, and for which you've already calculated and remembered the relevant totals A and B.

To force the outcome of the game, you follow a simple strategy for your moves:

Always choose the coin next to the one which your friend just took.

To help prevent this strategy from becoming too noticeable, you might want to once or twice start taking a coin from the opposite end and then, at the last minute, change your mind. Use your acting skills to make it seem as if this is a genuinely spontaneous choice!

WHY IT WORKS

Uthara goes first, and both of her choices are in odd-numbered positions. Whichever one she picks, the coin next to it will be at an even-numbered position. You take that coin, and now Uthara again has a choice of two odd-numbered coins. Continuing in this way, Uthara will end up with all the odd-numbered coins and you will end up with all the even-numbered ones.

THREE HEAPS AND AN IMPOSSIBILITY

The following routine can be done with borrowed objects (we'll use coins, but you could also use pebbles or candies, for example). It can even be done virtually over the internet, using a video chat platform such as Zoom. If you're performing this in person, you can turn your back throughout the routine so that it's clear you can't see anything your friend is doing.

1. Ask your friend to grab a bunch of coins (or candies, or any other small object). Tell her she should have at least 15 coins, but it could be as many as 40 or 45 if she wishes. (If performing this trick in person, you may wish to provide your friend with a stack of coins that you've brought along yourself.)

2. Tell your friend to make three heaps of coins on the table, *each with the same number of coins in it.* She does not need to use all of her coins, but each pile she makes should contain at least five coins. She should place one heap to her left, one in the middle, and one to her right, and she should not tell you how many coins are in each pile. Have her place any remaining coins in her pocket, or off to the side.

3. Instruct your friend to take four coins from the pile on the right and put them in the center pile. Then she should take four coins from the pile on the left and put them in the center pile. Point out that the center pile now has the most coins.

4. Ask your friend to count silently the number of coins in either the left or right pile; she does not need to tell you which pile or say what the total is. Have her remember this number.

5. Whatever number she just counted, ask her to remove that many coins from the center heap.

6. Ask her to place the coins she just removed onto either the left-hand or right-hand pile. (She does not need to tell you which). Point out that she now has three different-sized piles, and there's no way you could know how many coins are in each pile, since you don't even know how many coins she started with!

7. Ask her to place her hand onto the pile with the smallest number of coins in it. Then instruct her to move her hand either left or right onto whichever pile is closest.

8. Tell your friend that, despite the fact that *she* has made all the choices here, without telling you *anything*, you will tell her exactly how many coins are now underneath her hand. Confidently announce that there are precisely 12 coins beneath her hand. When she counts, she will discover that you are correct!

WHY IT WORKS

Suppose, at the beginning, that there are N coins in each of the three piles. Because we limited your friend to at most 45 coins in total, N can be no bigger than 45/3 = 15.

Let's call the piles A, B, and C, respectively (from left to right).

After Step 3 above, there will be N-4 coins in piles A and C and N+8 coins in pile B.

In Step 4, the spectator will always count the number N-4. When she removes this many coins from the middle heap in Step 5, there will be (N+8) - (N-4) = 12 coins remaining in Pile B. After Step 6, the smaller of the other two piles will still have N-4 coins in it.

Since N-4 is at most 15-4 = 11, the smallest pile will be either the one on the left or the one on the right. So in Step 7, your friend will place her hand on one of those two piles. When she moves her hand left or right to the closest pile, she is therefore guaranteed to end up on Pile B.

PREDICTING THE WINNER

The "contest" which follows appears to be a game of pure chance, but thanks to a clever mathematical principle you will be able to predict the outcome with perfect accuracy every time!

1. Show your friends Arnav and Khushi a paper bag (or any other receptacle that you can't see through) filled with green and red candies (or any other two small objects which feel the same but look different). Tell them that they'll be playing a game and you're going to try to predict the outcome.

2. Ask Arnav to choose whether he'd like to go for green or red.

3. If Arnav says "red," say, *"OK, so that leaves green for Khushi,"* and give Arnav your folded-up prediction to hold, saying that he'll get to read it out loud at the end. If he says "green," say, *"OK, so that leaves red for Khushi,"* and give **Khushi** your folded-up prediction to hold and read out loud at the end. (In other words, the prediction goes to whoever ends up with "red.")

4. Ask Khushi whether she wants to go first or second. Whoever is designated to go first, ask him or her to reach into the bag without looking and withdraw one candy.

5. Your friends now alternate picking out a single candy on each turn, without looking, until the bag is empty.

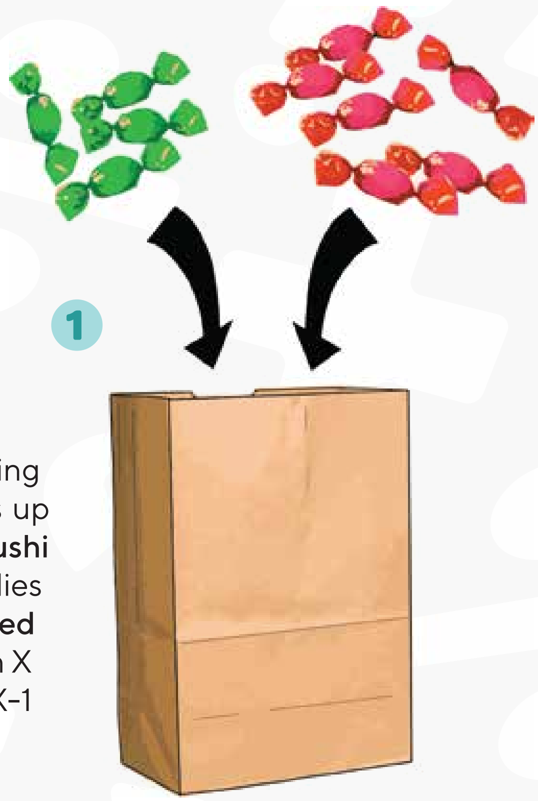
6. Ask your friends each to count how many of their color they received. Let's say Khushi is playing for red and she gets 3 red candies, while Arnav is playing for green and he gets 2 green candies. Summarize the outcome by saying, *"So that means that Khushi wins by one candy, right?"*

7. Ask the person holding the prediction to read it out loud. In this case, Khushi will read the prediction and it says, "I will win by one candy!"

WHY IT WORKS

The secret is that you start with two more red candies than green candies in the bag. For the game to be interesting but not take too long, you might wish, for example, to have 6 red and 4 green candies **(Illustration 1)**.

1



Amazingly, no matter how the players choose, your prediction will always be correct!

To explain why, we'll use a little bit of algebra. Let's say that we begin with N green candies and N+2 red candies in the bag, so that there are 2N+2 candies altogether and each player will end up with half, which is N+1.

Suppose Arnav is going for green and Khushi is going for red. The number of green candies Arnav ends up with is N minus the number of green candies **Khushi** ends up with. And the number of green candies Khushi ends up with is N+1 minus the number of **red** candies she ends up with. So if Khushi ends up with X red candies, Arnav ends up with N - (N+1 - X) = X-1 green candies. Therefore Khushi always wins by 1.

THE CARDS KNEW

This is a baffling and memorable trick using an ordinary shuffled deck of cards.

1. Remove the four Aces from a standard deck of fifty-two cards and place the Aces in a face-up pile on the side of the table. Have your friend shuffle the rest of the deck.

2. Separate the deck into a number of smaller piles of varying sizes and lay each of these piles face down onto the table, retaining just a few cards in your hand. (This should be done casually, but in fact there's a special procedure you must follow here which will be described below.)

3. Ask your friend to pick up the four Aces and place each Ace face up onto one of the face-down piles. Confirm that there's no way you could have known which four piles she would choose **(Illustration 1)**.

4. Collect all of the face-down piles without an Ace on top and place them onto the cards in your hand. Give this packet of cards to your friend to hold, and confirm that there's no way you could know how many cards she is holding.

5. Remove the Aces from the four piles on the table and turn over the top card of each pile. Say that while you have no way of knowing how many cards your friend is holding, the cards themselves know **(Illustration 2)**!

6. As you turn the top card of each pile face up, add the values of these cards together (Jacks, Queens, and Kings count as ten). For example, you might say, *"Seven, plus this five is 12, plus a ten makes 22, plus this two makes 24.* ***You*** *chose these four packets, and the total comes out to exactly 24."*

1

7. Say, *"There's no way I could have known which four packets you'd choose, or how many cards would be left over.* **But the cards knew!***"* Ask your friend to count how many cards she is holding. It will match the total just announced (24 in the above example) **(Illustration 3, next page)**.

HOW TO DO IT

2

The secret is that there's a special procedure for creating the face-down piles. With the deck in your left hand and facing you, note the value of the first card you see (again, face cards count as ten). Start counting upward from this value until you reach eleven, taking one card into your right hand each time you count, with each card going on *top* of the previous ones.

For example, if the face card of the deck is a seven, take the seven face up into your other hand and count, "7," then move the next face-up card on top of the seven and count, "8," then move another card, and so on, until you reach a count of "11." At this point, turn the packet of cards face down and place it in a pile onto the table.

In the above example, the packet would consist of five cards, since you count, "7... 8... 9... 10... 11." If the face card of the deck had been a four, you would have counted, "4... 5... 6... 7... 8... 9... 10... 11," and the packet would consist of eight cards.

Repeat this procedure, creating face-down packets of different sizes, until you no longer have enough cards to total eleven. Keep the remaining cards (if there are any) in your left hand.

WHY IT WORKS

The reason the trick works is that the number of cards in each face-down pile is *12 minus the value of the top card of the pile*. For example, when we started with a seven we ended up with a pile of five cards (note that 7+5=12), and when we started with a four we ended up with a pile of eight cards (and 4+8 is also 12).

It follows that *the total number of cards in the four face-down piles chosen by your friend will be 48 minus the sum of the values of the four top cards*. Indeed, if the values of the four top cards are A, B, C, and D, then the total number of cards in the four piles will be
$(12-A) + (12-B) + (12-C) + (12-D) = 48 - N$, where $N = A+B+C+D$.

Since we set aside the four Aces at the beginning and they're not involved in any of the counting, there are a total of 48 cards between the four face-down piles and the cards which your friend is holding. Since there are 48 - N cards in the four face-down piles, that means your friend is holding exactly N cards!

ANIMAL DOMINOES

This is a fun and interactive demonstration which has a surprisingly rich underlying mathematical explanation.

1. Punch out the ten animal dominoes and turn them all face down. Ask your friend to select one of the dominoes and place it to the side, face down, without looking at the animals on it. Tell your friend that this will be her secret prediction!

2. Ask your friend to turn the other nine dominoes face up and try to construct a continuous chain of dominoes stretching from one side of the table to the other in a straight line.

3. When she has finished creating a single chain, point out to your friend that she made a number of choices along the way and then ended up with this particular configuration. Also point out that there are now precisely two unmatched animals, one on each end of the chain, and ask her to imagine a domino with THOSE two animals on it.

4. Remind your friend that she made a prediction at the very beginning. Ask her to turn over the domino that she originally set aside: it will be the exact domino she was just visualizing in her mind!

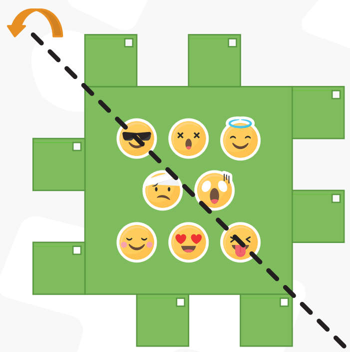
WHY IT WORKS

First, we claim that if we use ALL the dominoes (including the prediction) to form a single chain, the unmatched animals at two ends will always be the same. The reason is that the animals which are NOT at one of the two ends naturally come in pairs, and each animal is depicted on an EVEN number of dominos (namely, 4). If the animal on the left-hand side of the chain did not match the animal on the right-hand side, then each would appear an odd number of times in the chain, which is impossible.

Now suppose we do the same thing, but with one domino left out as our "prediction." The two animals on the missing domino belong to an ODD number of dominoes in the chain, while the rest belong to an even number. This time, the two dominoes at the end of the chain cannot match, since otherwise (reasoning as above) every animal would belong to an EVEN number of dominoes in the chain. So, by the same logic as before, the animals on the two ends of the chain are represented on an ODD number of dominoes (not counting the prediction). The only two animals with this property are the ones belonging to the missing domino.

BUILD YOUR OWN LIE DETECTOR

We're going to build a magical lie detecting machine together — using math! Just follow the steps below.



1. Cut out the master "Lie Detector" card and both sides of each of the seven colored cards with emojis on them. Then cut out the small white squares in each of the colored cards to make little square holes. Glue or tape the two sides of each colored card together, making sure to align all the square holes.

2. Ask your friend to think of one of the emojis on the Lie Detector.

3. Show your friend the face-up side of one of the colored cards and ask if she sees her thought-of emoji on that card. If she answers **yes**, place the card on the Lie Detector so that the side she looked at is showing. If she answers **no**, flip the card along its upper-left to lower-right diagonal axis so that the OTHER side is showing, and then place it on the Lie Detector.

4. Repeat with the other six colored cards. Tell your friend that most of the time she must tell the truth, but she should lie *exactly* once. Ask her to remember the color of the card she lied about.

5. Once all seven colored cards have been placed on the Lie Detector, you will find that there is a single white hole visible through one of the tabs. The emoji beneath this hole (on the Lie Detector card) is the one your friend was thinking of. And the color of the tab surrounding the hole is the color of the card your friend lied about!

WHY IT WORKS

This trick is based on a mathematical construction known as an *error-correcting code*.

Error-correcting codes are used in cell phones, DVD players, satellites, and many other pieces of modern technology. Their importance stems from the fact that if you transmit a string of 0s and 1s and part of the transmitted signal gets corrupted, the code is not only able to *detect* the error, it can *correct* it, too!

This is the exact same principle on which our Lie Detecting machine works. The particular code being used here is known as the "Hamming Code" (it was discovered by the mathematician Richard Hamming in the 1950s). There is a way to identify each of the seven cards in our magic trick with one of the seven "code words" in the Hamming Code, and the error-detecting property of this code translates to the ability of our "Lie Detecting machine" to identify a single falsehood.

BE A HUMAN CALCULATOR!

With this impressive demonstration, you will be able to convince your friends that you can do lightning-fast superhuman mental calculations. In fact, though, there's a devious secret which makes it very easy to do!

1. Cut out the five unfolded dice on the following pages and fold each one into a three-dimensional cube. Use glue or double-stick tape to adhere the tabs inside the cube.

2. Give the dice to your friend and ask her to roll them, saying that you will try to add together the five three-digit numbers that she rolls as quickly as possible.

3. In just a split second, as soon as she rolls the dice you are able to tell her the total of all five numbers! She can then use a calculator to verify that you are 100% correct.

4. You can repeat the demonstration again if you wish!

HOW TO DO IT

The sum of the five numbers will always be a four-digit number N. Here's the shortcut for figuring out what N is:

Add together the final digit of the numbers on each of the five dice. The total X will tell you the final two digits of N.

Subtract X from 60. The result Y will tell you the first two digits of N.

For example, suppose your friend rolls 545 on the yellow die, 954 on the red die, 469 on the green die, 973 on the blue die, and 386 on the brown die. You ignore all but the final (ones) digits 5, 4, 9, 3, and 6, respectively, and simply add these numbers up in your head. This gives $X = 5 + 4 + 9 + 3 + 6 = 27$. So the last two digits of N are 2 and 7. Subtracting 27 from 60 gives Y=33, so the first two digits of N are 3 and 3. In other words, the total of the five dice is 3327.

Go ahead, check it on a calculator! Are you stumped as to how this works even though we just explained it to you?

WHY IT WORKS

Here are the two-digit numbers appearing on the five dice:

Yellow: 545, 644, 743, 347, 446, 248
Red: 756, 855, 459, 657, 558, 954
Green: 766, 865, 667, 964, 568, 469
Blue: 874, 676, 577, 775, 478, 973
Brown: 683, 584, 287, 188, 386, 485

Let's first concentrate just on the last two digits of the numbers on the five dice. A hidden pattern emerges if we list these two-digit numbers in order:

Yellow: 43, 44, 45, 46, 47, 48
Red: 54, 55, 56, 57, 58, 59
Green: 64, 65, 66, 67, 68, 69
Blue: 73, 74, 75, 76, 77, 78
Brown: 83, 84, 85, 86, 87, 88

If we now focus on the first and last digits of the numbers on each die, we discover another hidden pattern: they always sum to 10 on the yellow die, to 13 on the red die, to 13 on the green die, to 12 on the blue die, and to 9 on the brown die.

This means we can write the three-digit numbers on the dice as follows:

Yellow: $a'4a$ Red: $b'5b$ Green: $c'6c$ Blue: $d'7d$ Brown: $e'8e$

where $a' = 10 - a$, $b' = 13 - b$, $c' = 13 - c$, $d' = 12 - d$, $e' = 9 - e$.

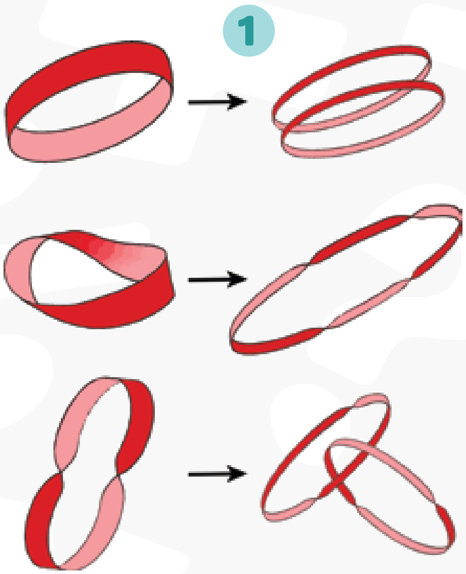
The sum of the five three-digit numbers which the spectator rolls is therefore

$$(100a' + 40 + a) + (100b' + 50 + b) + (100c' + 60 + c) + (100d' + 70 + d) + (100e' + 80 + e)$$
$$= 100*(a' + b' + c' + d' + e') + (40 + 50 + 60 + 70 + 80) + (a + b + c + d + e)$$
$$= 100*(10 + 13 + 13 + 12 + 9 - (a+b+c+d+e)) + 300 + (a + b + c + d + e)$$
$$= 100*(57 - (a+b+c+d+e)) + 100*3 + (a + b + c + d + e)$$
$$= 100*(57 + 3 - (a + b + c + d + e)) + (a + b + c + d + e)$$
$$= 100*(60 - (a + b + c + d + e)) + (a + b + c + d + e)$$
$$= 100*Y + X.$$

Note that since a and d are between 3 and 8 inclusive, b and c are between 4 and 9 inclusive, and e is between 3 and 8 inclusive, we find that $a + b + c + d + e$ is between 17 and 42 inclusive (since $17 = 3 + 3 + 4 + 4 + 3$ and $42 = 8 + 8 + 9 + 9 + 8$). In particular, both $X = a + b + c + d + e$ and $Y = 60 - (a + b + c + d + e)$ are two-digit numbers.

TWISTED BANDS

This is one of the most famous math-based magic tricks and it's based on the branch of mathematics known as topology.



More specifically, this trick utilizes the *Möbius strip*, a twisted 3-dimensional shape with some remarkable properties that you'll explore for yourself in the process of practicing and performing. You will need two pairs of scissors and a large sheet of wrapping paper or newsprint in order to perform this effect for an audience, although you can practice it yourself with just one pair of scissors and a smaller-sized piece of paper. Optionally, you may also offer a "prize" for the winner of the contest you're about to hold (don't worry, no one is going to win the prize, thanks to the power of mathematics!).

1. Cut out three long paper strips from the paper, approximately 10-20 centimeters wide and between 1.5 and 3.5 meters long. (The exact dimensions are quite flexible, though, so use whatever you have; the larger the strips, the less noticeable the twists you're about to put in them will be.)

2. Using tape, glue, or rubber cement, connect the ends of each of the three strips as follows:

a) In Strip A, connect the ends without twisting to form a circular band.

b) In Strip B, give the strip a half-twist (i.e., a 180-degree twist) before taping or pasting the ends together.

c) In Strip C, give the strip two half-twists (i.e., a full 360-degree twist) before taping or pasting the ends together.

3. Call up two volunteers from your audience — let's suppose their names are Bharati and Chandra — and hand Strip B to Bharati and Strip C to Chandra. Keep Strip A for yourself.

4. Show Bharati and Chandra the prize that they'll be competing for, and then explain how the contest will work. Tell them that it's a race to see who can cut their band into two separate pieces first. Demonstrate by cutting your own band in half, cutting lengthwise along the center until you've cut all the way around and return to your starting point. You are now holding two thinner bands, which you display to your audience (one in the left hand, one in the right).

5. Hand a pair of scissors to each of your volunteers and tell them to get ready. When you say "Go!" they begin cutting.

6. Whichever of the volunteers finishes first (let's suppose it's Bharati), begin to hand her the prize, but then suddenly notice that she has not correctly followed your instructions: she has produced a **single** large ring rather than two separate rings. Offer the prize to Chandra instead, only to notice that he has likewise failed to follow the instructions: he has produced two **linked** bands **(Illustration 1)**!

WHY IT WORKS

As we mentioned earlier, the trick relies on the topological properties of bands with twists in them. Strip B, after the cutting, will be twice as long as the original strip and have **four** half-twists in it, while Strip C will consist of two linked loops, each with two half-twists.

It's actually quite difficult to explain **why** it works, but we'll try: Because of the half-twist, the original Strip B only has **one** edge, which is twice as long as the original strip. Cutting Strip B down the middle creates a second edge of the same length, half on each side of the scissors, resulting in one twisted strip that is twice as long as the original one. It's a bit harder to explain why Strip C ends up with two linked pieces, so we'll leave that one to your imagination!

In case you were wondering, a strip with three half-twists, when bisected lengthwise as we did with Strips B and C, becomes a twisted strip tied in what mathematicians call a **trefoil knot**. (Try it!) If this knot is unraveled, it will be found to contain eight half-twists.

What happens if you put even **more** twists into a band and then cut it down the center? The answer depends on whether the number of twists — call it n — is even or odd:

If n is even, you get two interlocking bands, each with n half-twists.
If n is odd, you get a single loop with $2n+2$ half-twists.

CREDITS

Every magic trick has to be invented by someone. Where possible, we asked permission from the people who invented the tricks in Math Miracles:

Fingertip Mindreading
Master magician Jim Steinmeyer is the creator of "Fingertip Mindreading." This trick, along with many other great mathematical miracles, can be found in *Subsequent Impuzzibilities*.

Follow the Money
This wonderful effect was created by magician Bob Hummer, who called it "Mathematical Monte."

Animal Imagery
The creator of this sequence is unfortunately lost to time, but our inspiration was a version published as "Where in the World?" from *Magic Scratchers* by the magician Danny Orleans.

1240 Puzzle
Our friend Richard Wiseman is a devious person. He created this puzzle.

Making Someone Vanish
Martin Gardner was a genius magician and mathematician. He took a geometric puzzle dating back to at least the mid-1500s and turned it into this fantastic idea.

Which Hand?

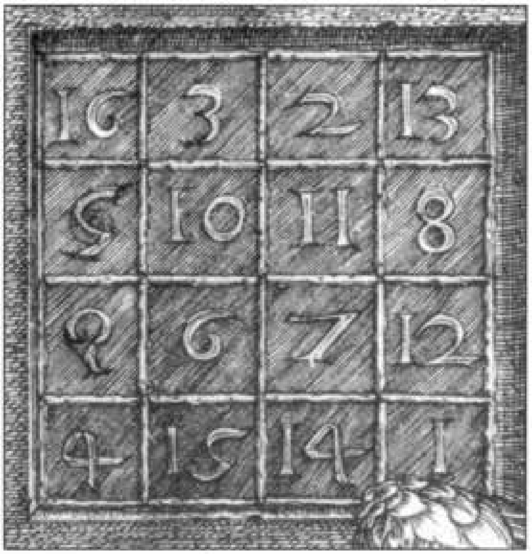
Magicians have used logic puzzles as secret methods for tricks for many years. Welsh magician Mark Elsdon popularized using a logic puzzle for a "which hand" puzzle. Matt Baker created this particular version, however.

Mysterious Magic Square

There are many different types of magic squares, and mathematicians have studied them for thousands of years. The oldest known 4x4 magic square comes from India, in a work by Varahamihira called the *Brhat Samhita*, which was written more than 1400 years ago (in the year 587). Magic Squares have been used throughout Indian history for divination, star-gazing, and even choosing ingredients for perfumes!

The particular magic square we've chosen for this trick, in the special case N=34, was used by the artist Albrecht Dürer in his famous engraving entitled "*Melancolia I.*" The two numbers in the middle of the bottom row give the date of the engraving, 1514 (and yes, Dürer did that on purpose).

From Dürer's "Melancolia I" (source: Wikipedia)



It is possible to make bigger magic squares (such as a 5 x 5 square), and even magic squares out of other shapes! The one we teach here provides a relatively simple method for creating a magic square quickly, since only four numbers change each time you do it.

Missing Digit

Sam Loyd was one of the early utilizers of this principle, and a similar trick appears in Harry Lorayne's *The Magic Book*. However, in Lorayne's version the procedure is 'deterministic' (Principle #1 is not used).

Predicted Change

The math principle underlying this trick was published in Peter Winkler's book *Mathematical Puzzles: A Connoisseur's Collection*. It's not due to Peter Winkler, though; it was passed along to him by the Israeli mathematician Ehud Friedgut, and was apparently used before that by a high-tech company in Israel to test job candidates.

The coin trick we've presented here is a simplified version of a routine created by Michael Weber; we thank Michael for his permission to include "Predicted Change" in this book.

Three Heaps and an Impossibility

This is Jim Steinmeyer's "Welcome Change", from his book *Virtual Impuzzibilities*. It is based on "The Three Heaps" from Martin Gardner's *Mathematics, Magic, and Mystery*.

Predicting the Winner

The first applications of the "red-black relationship principle" (as the principle underlying this trick is known in magic circles) to magic appear to be Stewart James' "Tapping a Brain Wave" and "The Psychic Pickpocket," both created in 1938 but not published until 2000, and Oscar Weigle's "The Little Star Prediction," published in 1939. The direct inspiration for "Predicting the Winner" was Matt Baker's "Hive Mind," from his book *The Buena Vista Shuffle Club*.

The Cards Knew

The trick and handling here are adapted directly from Richard Vollmer's "The Cards Knew," which was published in Roberto Giobbi's book *Card College Lighter*. Vollmer's routine was adapted from Dai Vernon's "Affinities," which appeared in *The Vernon Chronicles, Volume 2*. Vernon's routine is based upon a very old mathematical principle (origin unknown).

Animal Dominoes

The domino prediction appears as "The Break in the Chain" in Martin Gardner's 1956 book *Mathematics, Magic & Mystery*, but the idea long predates Gardner. According to Max Maven, it can be found at least as far back as Hoffmann's *Modern Magic* (1876).

Build Your Own Lie Detector

The idea to build the cards with these kinds of protruding tabs comes from Richard Ehrenborg ("Decoding the Hamming Code", Volume 13, Issue 4, 2006 of *Math Horizons*). Ehrenborg's system was improved to include the small square holes by Todd Mateer ("A Magic Trick Based on the Hamming Code", Volume 21, Issue 2, 2013 of *Math Horizons*). We have adapted our design of the cards and tabs from Mateer's paper.

Be a Human Calculator!

Like many tricks in this book, the idea started with mathematician, author, and magician Martin Gardner.

Twisted Bands

This trick is sometimes called Afghan Bands and became a popular magic trick in the late 1800s. The name "Afghan Bands" for this trick was coined by P.T. Selbit in 1901 (Selbit was also the first magician to perform the famous illusion of sawing a woman in half). The routine we have described here was developed by Phil Foxwell, as described in Martin Gardner's classic book *Mathematics, Magic, and Mystery*.

The Möbius strip is named after the German mathematician August Ferdinand Möbius. It was independently discovered by Möbius and Johann Benedict Listing in the 1850s, but similar shapes can be seen in mosaics dating back to ancient Rome.

MORE MAGIC!

If you're as fascinated as we are by magic and math and the places where the two intersect, we have three suggestions for you.

1. Check out "Magical Mathematics: The Mathematical Ideas That Animate Great Magic Tricks" by Persi Diaconis and Ron Graham. The math involved is more complex than the tricks described here, but advanced students will love the principles the authors explore.

2. Join a magic club! The Society of Young Magicians has online meetings where you can learn and perform magic with other young wizards. Info is available at **www.magicsam.com**

3. Learn more magic! You can learn many new tricks and pick up essential magic supplies at Vanishing Inc. Magic: **www.vanishingincmagic.com**